Cambridge O Level

Cambridge Assessment International Education
Cambridge Ordinary Level

ADDITIONAL MATHEMATICS **4037/12**

Paper 1 **May/June 2018**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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Cambridge Assessment International Education

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

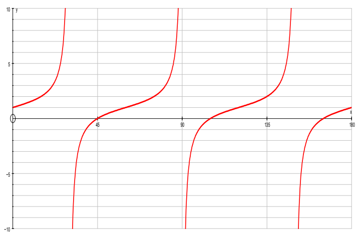
A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Partial Marks
1(i)	$\frac{\pi}{3}$ or 60°	**B1**	
1(ii)		**3**	**B1** for 3 asymptotes at $x=30^{\circ}$, 90° and 150°; the curve must approach but not cross all 3 of the asymptotes and be in the 1st and 4th quadrants **B1** for starting at $(0, 1)$ and finishing at $(180, 1)$ **B1** for all correct
2	For an attempt to obtain an equation in x only	**M1**	
	$9x^2-(k+1)x+4=0$	**A1**	correct 3 term equation
	$(k+1)^2-(4\times 9\times 4)$	**M1**	**M1dep** for correct use of b^2-4ac oe
	Critical values $k=11$, $k=-13$	**A1**	
	$-13<k<11$	**A1**	For the correct range
3	$e^y=ax^2+b$	**B1**	may be implied, $b\neq 0$
	either $3=5a+b$ $1=3a+b$ or Gradient = 1, so $a=1$	**M1**	correct attempt to find a or b by use of simultaneous equations or finding the gradient and equating it to a
	Coefficient of x^2 is 1	**A1**	
	Intercept is –2	**A1**	
	$y=\ln(x^2-2)$	**A1**	For correct form
4(i)	$3=\ln(5t+3)$ $e^3=5t+3$ or better	**B1**	
	$t=3.42$	**B1**	
4(ii)	$\frac{dx}{dt}=\frac{5}{5t+3}$	**M1**	for $\frac{k_1}{5t+3}$
	When $t=0$, $\frac{dx}{dt}=\frac{5}{3}$, 1.67 or better	**A1**	all correct

Question	Answer	Marks	Partial Marks
4(iii)	If $t > 0$ each term in $\frac{k_1}{5t+3} > 0$ so never negative oe	B1	**dep** on **M1** in **(ii)** **FT** on *their* $\frac{k_1}{5t+3}$, provided $k_1 > 0$
4(iv)	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = \frac{k_2}{(5t+3)^2}$	M1	
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\frac{25}{(5t+3)^2}$ When $t=0$, $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\frac{25}{9}$ or -2.78	A1	all correct
5(i)	$a = 243,\ b = -45,\ c = \frac{10}{3}$	3	**B1** for each coefficient, must be simplified
5(ii)	$\left(243 - \frac{45}{x} + \frac{10}{3x^2}\right)\left(4 + 36x + 81x^2\right)$	B1	For $\left(4 + 36x + 81x^2\right)$
	for having 3 terms independent of x	M1	
	Independent term is $972 - 1620 + 270 = -378$	A1	
6	attempt to differentiate quotient or equivalent product	M1	
	$\frac{\mathrm{d}}{\mathrm{d}x}(2x-1)^{\frac{1}{2}} = (2x-1)^{-\frac{1}{2}}$ for a quotient $\frac{\mathrm{d}}{\mathrm{d}x}(2x-1)^{-\frac{1}{2}} = -(2x-1)^{-\frac{3}{2}}$ for a product	B1	
	either $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sqrt{2x-1} - (x+2)\left[(2x-1)^{-\frac{1}{2}}\right]}{\left(\sqrt{2x-1}\right)^2}$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = (2x-1)^{-\frac{1}{2}} - (x+2)\left[(2x-1)^{-\frac{3}{2}}\right]$	A1	All other terms correct
	When $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$, $2x - 1 = x + 2$	M1	equate to zero and attempt to solve
	$x = 3$	A1	
	$y = \sqrt{5}$, $\frac{5}{\sqrt{5}}$, 2.24	A1	

Question	Answer	Marks	Partial Marks
7(i)	1000	**B1**	
7(ii)	$2000 = 1000e^{\frac{t}{4}}$	**B1**	
	$t = 4\ln 2,\quad \ln 16$	**M1**	For $4\ln k$ or $\ln k^4$, $k > 0$
	2.77	**A1**	
7(iii)	$B = 1000e^2$ $= 7389, 7390$	**B1**	
8(a)	$3\left(1-\sin^2\theta\right)+4\sin\theta = 4$	**M1**	use of correct identity
	$(3\sin\theta-1)(\sin\theta-1)=0$ $\sin\theta = \frac{1}{3},\quad \sin\theta = 1$	**M1**	For attempt to solve a 3 term quadratic equation in $\sin\theta$ to obtain $\sin\theta =$
	$\theta = 19.5^\circ,\ 160.5^\circ$	**A1**	
	90°	**A1**	
8(b)	$\tan 2\phi = \sqrt{3}$ $2\phi = \frac{\pi}{3},\ -\frac{2\pi}{3}$	**M1**	obtaining an equation in $\tan 2\phi$ and correct attempt to solve for one solution to reach $2\phi = k$
	for one correct solution $\phi = \frac{\pi}{6}$, or 0.524	**A1**	
	for attempt at a second solution	**M1**	
	$\phi = -\frac{\pi}{3}$, or -1.05	**A1**	for a correct second solution and no other solutions within the range
9(a)(i)	1000	**B1**	
9(a)(ii)	for use of power rule	**M1**	
	for addition or subtraction rule	**M1**	**dep** on previous **M1**
	$\lg\frac{1000a}{b^2}$	**A1**	Allow $\lg\frac{10^3 a}{b^2}$
9(b)(i)	$x^2 - 5x + 6 = 0$	**M1**	For attempt to obtain a quadratic equation and solve
	$x = 3,\ x = 2$	**A1**	for both

Question	Answer	Marks	Partial Marks
9b(ii)	$(\log_4 a)^2 - 5\log_4 a + 6 = 0$	M1	For the connection with **(i)** and attempt to deal with at least one logarithm correctly, either $4^{their 3}$ or $4^{their 2}$
	$a = 64$	A1	
	$a = 16$	A1	
10(i)	$AC^2 = \left(4\sqrt{3}-5\right)^2 + \left(4\sqrt{3}+5\right)^2$	M1	For attempt to use the cosine rule
	$-2\left(4\sqrt{3}-5\right)\left(4\sqrt{3}+5\right)\cos 60^\circ$	A1	For all correct unsimplified
	$AC^2 = 123$	M1	**M1 dep** for attempt to evaluate without use of calculator
	$AC = \sqrt{123}$	A1	
	ALTERNATIVE METHOD		
	Taking D as the foot of the perpendicular from A: Find AD, BD, DC $AC^2 = AD^2 + DC^2$	M1	For a complete method to get AC^2
	$AC^2 = \left(\frac{12-5\sqrt{3}}{2}\right)^2 + \left(\frac{15+4\sqrt{3}}{2}\right)^2$	A1	For all correct unsimplified
	$AC^2 = 123$	M1	**M1dep** for attempt to evaluate without use of calculator
	$AC = \sqrt{123}$	A1	

Question	Answer	Marks	Partial Marks
10(ii)	$\frac{AC}{\sin 60^{\circ}} = \frac{4\sqrt{3}-5}{\sin ACB}$ or $\sin ACB = \frac{AD}{AC}$	M1	For attempt at the sine rule or trigonometry involving right-angled triangles
	For attempt at cosec	M1	**dep** on first **M** mark $\text{cosec}ACB = \frac{2\sqrt{123}}{\sqrt{3}\left(4\sqrt{3}-5\right)}$ or $\frac{2\sqrt{41}}{\left(4\sqrt{3}-5\right)}$ oe
	$\text{cosec } ACB = \frac{2}{\sqrt{3}}\frac{\sqrt{123}}{\left(4\sqrt{3}-5\right)} \times \frac{4\sqrt{3}+5}{4\sqrt{3}+5}$	M1	**dep** on previous **M** mark for a statement involving rationalisation using $a\sqrt{3}+b$
	$= \frac{2\sqrt{41}}{23}\left(4\sqrt{3}+5\right)$	A1	For rationalisation using $\frac{4\sqrt{3}+5}{4\sqrt{3}+5}$ oe and simplification
	ALTERNATIVE METHOD		
	$\frac{1}{2}\left(4\sqrt{3}-5\right)\left(4\sqrt{3}+5\right)\sin 60 \quad = \frac{23\sqrt{3}}{4}$	M1	Area of ABC
	$\frac{1}{2}\sqrt{123}\left(4\sqrt{3}+5\right)\sin ACB \quad = \frac{23\sqrt{3}}{4}$	M1	For attempt at a second area of ABC and equating to first area
	For attempt at cosec	M1	**dep** on first 2 **M** marks
	$= \frac{2\sqrt{41}}{23}\left(4\sqrt{3}+5\right)$	A1	Need to be convinced no calculator is being used in simplification

Question	Answer	Marks	Partial Marks
11	When $x=0,\ y=\frac{1}{2}$	B1	For $y=\frac{1}{2}$
	$\frac{dy}{dx}=\frac{1}{2}e^{4x}$	B1	
	$\frac{dy}{dx}=\frac{1}{2}$, Gradient of normal = –2	B1	**FT** on *their* $\frac{dy}{dx}$, must be numeric
	either: Normal $y-\frac{1}{2}=-2x$ or: Gradient of normal = $-\frac{OA}{OB}$	M1	For an attempt at a normal equation passing through *their* $\left(0,\frac{1}{2}\right)$ and a substitution of $y=0$
	When $y=0,\ x=\frac{1}{4}$	A1	
	EITHER: $\int_0^{\frac{1}{4}}\frac{1}{8}e^{4x}+\frac{3}{8}dx$	M1	For attempt to integrate to obtain $k_1e^{4x}+\frac{3}{8}x,\ k_1\neq\frac{1}{8},\ k_1\neq\frac{1}{2}$
	$\left[\frac{1}{32}e^{4x}+\frac{3x}{8}\right]_0^{\frac{1}{4}}$	A1	For correct integration
	Use of limits	M1	**M1dep**
	For area of triangle $=\frac{1}{16}$	B1	**FT** on *their* $x=\frac{1}{4}$
	$=\frac{e}{32}$	A1	final answer in correct form
	OR: $\int_0^{\frac{1}{4}}\frac{1}{8}e^{4x}+\frac{3}{8}-\frac{1}{2}+2x\ dx$	M1	For attempt at subtraction and attempt to integrate to obtain $k_1e^{4x}+\frac{3}{8}x+k_2x+k_3x^2,\ k_1\neq\frac{1}{8}$
	$\left[\frac{1}{32}e^{4x}-\frac{1}{8}x+x^2\right]_0^{\frac{1}{4}}$	A2	–1 for each error for integration
	for use of limits	M1	**M1dep**
	$=\frac{e}{32}$	A1	final answer in correct form

Question	Answer	Marks	Partial Marks
12(a)	$p = \frac{1}{4}$	B1	
	$p + q - 4q + 6 = 4$	B1	**FT** on *their p*
	$q = \frac{3}{4}$	B1	
12(b)	$\left(x^{\frac{1}{3}} + 3\right)\left(x^{\frac{1}{3}} + 1\right) = 0$	M1	For attempt to factorise and solve, or solve using the quadratic formula oe, a quadratic in $x^{\frac{1}{3}}$ or u
	$x^{\frac{1}{3}} = -1$ or $u = -1$ $x^{\frac{1}{3}} = -3$ or $u = -3$	A1	For both
	$x = -1$	A1	
	$x = -27$	A1	